

TOPIC 6

INDEFINITE & DEFINITE INTEGRALS

SCHEMATIC DIAGRAM

Topics	Concept	Degree of Importance	References
			Text book of NCERT, Vol. II 2007 Edition
Indefinite Integrals	(i) Integration by substitution	*	Exp 5&6 Page301,303
	(ii) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
	(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px + q)dx}{ax^2 + bx + c}$, $\int \frac{(px + q)dx}{\sqrt{ax^2 + bx + c}}$	***	Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23
	(iv) Integration using Partial Fraction	***	Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
	(v) Integration by Parts	**	Exp 18,19&20 Page 325
	(vi) Some Special Integrals $\int \sqrt{a^2 \pm x^2} dx$, $\int \sqrt{x^2 - a^2} dx$	***	Exp 23 &24 Page 329
	(vii) Miscellaneous Questions	***	Solved Ex.41
Definite Integrals	(i) Definite Integrals based upon types of indefinite integrals	*	Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9
	(ii) Definite integrals as a limit of sum	**	Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
	(iii) Properties of definite Integrals	***	Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
	(iv) Integration of modulus function	**	Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

SOME IMPORTANT RESULTS/CONCEPTS

<p>* $\int x^n dx = \frac{x^{n+1}}{n+1} + c$</p> <p>* $\int 1 \cdot dx = x + c$</p> <p>* $\int \frac{1}{x^n} dx = -\frac{1}{x^{n-1}} + c$</p> <p>* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$</p> <p>* $\int \frac{1}{x} dx + c$</p> <p>* $\int e^x dx = e^x + c$</p> <p>* $\int a^x dx = \frac{a^x}{\log a} + c$</p> <p>* $\int \sin x dx = -\cos x + c$</p> <p>* $\int \sin x dx = -\cos x + c$</p> <p>* $\int \cos x dx = \sin x + c$</p> <p>* $\int \sec^2 x dx = \tan x + c$</p> <p>* $\int \operatorname{cosec}^2 x dx = -\cot x + c$</p> <p>* $\int \sec x \cdot \tan x dx = \sec x + c$</p> <p>* $\int \operatorname{cosec} x \cdot \cot x dx = -\operatorname{cosec} x + c$</p> <p>* $\int \tan x dx = -\log \cos x + c = \log \sec x + c$</p> <p>* $\int \cot x dx = \log \sin x + c$</p> <p>* $\int \sec x dx = \log \sec x + \tan x + c$</p> <p style="text-align: center;">$= \log\left \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right + c$</p> <p>* $\int \operatorname{cosec} x dx = \log \operatorname{cosec} x - \cot x + c$</p> <p style="text-align: center;">$= -\log \operatorname{cosec} x + \cot x + c$</p> <p>$= \log\left \tan\frac{x}{2}\right + c$</p> <p>* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left \frac{x-a}{x+a}\right + C, \text{ if } x > a$</p> <p>* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left \frac{a+x}{a-x}\right + C, \text{ if } x > a$</p> <p>* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left \frac{a+x}{a-x}\right + C, \text{ if } x > a$</p>	<p>* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \frac{x}{a} + C$</p> <p>* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log x + \sqrt{x^2 + a^2} + C$</p> <p>* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log x + \sqrt{x^2 - a^2} + C$</p> <p>*</p> <p>$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log x + \sqrt{x^2 + a^2} + C$</p> <p>*</p> <p>$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log x + \sqrt{x^2 - a^2} + C$</p> <p>* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$</p> <p>* $\int \{f_1(x) \pm f_2(x) \pm \dots \pm f_n(x)\} dx$</p> <p style="text-align: center;">$= \int f_1(x) dx \pm \int f_2(x) dx \pm \dots \pm \int f_n(x) dx$</p> <p>* $\int \lambda f(x) dx = \lambda \int f(x) dx + C$</p> <p>* $\int u \cdot v dx = u \cdot \int v dx - \int \left[\frac{du}{dx} \cdot \int v dx \right] dx$</p> <p>$\int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx$</p> <p style="text-align: center;">* General Properties of Definite Integrals.</p> <p>* $\int_a^b f(x) dx = \int_a^b f(t) dx$</p> <p>* $\int_a^b f(x) dx = - \int_b^a f(x) dx$</p> <p>* $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$</p> <p>* $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$</p> <p>* $\int_0^a f(x) dx = \int_0^a f(a-x) dx$</p> <p>* $\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$</p>
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$* \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, = -\frac{1}{a} \cot^{-1} \frac{x}{a} + C$	$* \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$
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Assignments

(i) Integration by substitution

LEVEL I

1. $\int \frac{\sec^2(\log x)}{x} dx$

2. $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

3. $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

LEVEL II

1. $\int \frac{1}{\sqrt{x+x}} dx$

2. $\int \frac{1}{x\sqrt{x^6-1}} dx$

3. $\int \frac{1}{e^x-1} dx$

LEVEL III

1. $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx$

2. $\int \frac{\tan x}{\sec x + \cos x} dx$

3. $\int \frac{1}{\sin x \cdot \cos^3 x} dx$

(ii) Application of trigonometric function in integrals

LEVEL I

1. $\int \sin^3 x \cdot dx$

2. $\int \cos^2 3x \cdot dx$

3. $\int \cos x \cdot \cos 2x \cdot \cos 3x \cdot dx$

LEVEL II

1. $\int \sec^4 x \cdot \tan x \cdot dx$

2. $\int \frac{\sin 4x}{\sin x} dx$

LEVEL III

1. $\int \cos^5 x \cdot dx$

2. $\int \sin^2 x \cdot \cos^3 x \cdot dx$

(iii) Integration using standard results

LEVEL I

1. $\int \frac{dx}{\sqrt{4x^2-9}}$

2. $\int \frac{1}{x^2+2x+10} dx$

3. $\int \frac{dx}{9x^2+12x+13}$

LEVEL II

1. $\int \frac{x}{x^4+x^2+1} dx$

2. $\int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$

3. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

LEVEL III

1. $\int \frac{2x}{\sqrt{1-x^2-x^4}} dx$

2. $\int \frac{x^2+x+1}{x^2-x+1} dx$

3. $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

4. $\int \sqrt{\frac{1-x}{1+x}} dx$

5. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$ [CBSE 2011]

(iv) Integration using Partial Fraction

LEVEL I

1. $\int \frac{2x+1}{(x+1)(x-1)} dx$

2. $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$

3. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$

LEVEL II

1. $\int \frac{x^2+2x+8}{(x-1)(x-2)} dx$

2. $\int \frac{x^2+x+1}{x^2(x+2)} dx$

3. $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

LEVEL III

1. $\int \frac{8}{(x+2)(x^2+4)} dx$

2. $\int \frac{dx}{\sin x + \sin 2x}$

3. $\int \frac{1}{1+x^3} dx$

(v) Integration by Parts

LEVEL I

1. $\int x \cdot \sec^2 x \cdot dx$

2. $\int \log x \cdot dx$

3. $\int e^x (\tan x + \log \sec x) dx$

LEVEL II

1. $\int \sin^{-1} x \cdot dx$

2. $\int x^2 \cdot \sin^{-1} x \cdot dx$

3. $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}} dx$

4. $\int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \cdot dx$

5. $\int \sec^3 x \cdot dx$

LEVEL III

1. $\int \cos(\log x) dx$

2. $\int \frac{e^x(1+x)}{(2+x)^2} dx$

3. $\int \frac{\log x}{(1+\log x)^2} dx$

4. $\int \frac{2+\sin x}{1+\cos 2x} e^x \cdot dx$

5. $\int e^{2x} \cdot \cos 3x \cdot dx$

(vi) Some Special Integrals

LEVEL I

1. $\int \sqrt{4+x^2} \cdot dx$

2. $\int \sqrt{1-4x^2} \cdot dx$

LEVEL II

1. $\int \sqrt{x^2+4x+6} \cdot dx$

2. $\int \sqrt{1-4x-x^2} \cdot dx$

LEVEL III

$$1. \int (x+1)\sqrt{1-x-x^2} \cdot dx \qquad 2. \int (x-5)\sqrt{x^2+x} \, dx$$

(vii) Miscellaneous Questions

LEVEL II

$$1. \int \frac{1}{2-3\cos 2x} \, dx \qquad 2. \int \frac{1}{3+\sin 2x} \, dx \qquad 3. \int \frac{dx}{4\sin^2 x + 5\cos^2 x}$$

$$4. \int \frac{dx}{1+3\sin^2 x + 8\cos^2 x} \qquad 5. \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} \, dx \qquad 6. \int \frac{\sec x}{5\sec x + 4\tan x} \, dx$$

LEVEL III

$$1. \int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} \, dx \qquad 2. \int \frac{dx}{1-\tan x} \qquad 3. \int \frac{x^4}{x^4-1} \, dx$$

$$4. \int \frac{x^2+1}{x^4+x^2+1} \, dx \qquad 5. \int \frac{x^2-1}{x^4+1} \, dx \qquad 6. \int \sqrt{\tan x} \cdot dx$$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

$$1. \int_0^1 \frac{2x+3}{5x^2+1} \, dx \qquad 2. \int_0^{\pi/2} \sqrt{\sin x} \cdot \cos^5 x \, dx \qquad 3. \int_0^2 x\sqrt{x+2} \, dx$$

LEVEL II

$$1. \int_1^2 \frac{5x^2}{x^2+4x+3} \, dx \qquad 2. \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} \, dx$$

(ii) Definite integrals as a limit of sum

LEVEL I

$$1. \text{ Evaluate } \int_0^2 (x+2) \, dx \text{ as the limit of a sum.}$$

$$2. \text{ Evaluate } \int_0^4 (1+x) \, dx \text{ definite integral as the limit of a sum.}$$

LEVEL II

1. Evaluate $\int_1^2 (3x^2 - 1) dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2 + 1) dx$ as the limit of a sum.

LEVEL III

1. Evaluate $\int_1^2 (x^2 + x + 2) dx$ as the limit of a sum.

2. Evaluate $\int_2^4 (e^{2x} + x^2) dx$ as the limit of a sum.

(iii) Properties of definite Integrals

LEVEL I

1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx$

2. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$

3. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

LEVEL II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

2. $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

3. $\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \cos ec} dx$

4. $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ [CBSE 2011]

LEVEL III

1. $\int_0^{\pi} \frac{x + \sin x}{1 + \cos x} dx$ [CBSE 2011]

2. $\int_0^{\pi/2} \log \sin x dx$

3. $\int_0^{\pi/4} \log(1 + \tan x) dx$

[CBSE 2011]

(iv) Integration of modulus function

LEVEL III

1. $\int_2^5 (|x-2| + |x-3| + |x-4|) dx$

2. $\int_{-1}^2 |x^3 - x| dx$

3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|] dx$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2-3x-18}$

2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x \cdot dx$

5. Evaluate $\int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x} \cdot \cos^5 x \cdot dx$

9. Evaluate $\int_0^{\pi/2} \log \sin x \cdot dx$

4. Evaluate $\int \frac{dx}{3 + 2 \sin x + \cos x}$

6. Evaluate $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}} dx$

8. Evaluate $\int_{-1}^{3/2} |x \sin \pi x| dx$

10. Evaluate $\int_1^4 (|x-1| + |x-2| + |x-3|) dx$

Answers

TOPIC 6 INDEFINITE & DEFINITE INTEGRALS

(i) Integration by substitution

LEVEL I 1. $\tan(\log_e x) + C$ 2. $\frac{1}{m} e^{m \tan^{-1} x} + C$ 3. $e^{\sin^{-1} x} + C$

LEVEL II 1. $2 \log_e |1 + \sqrt{x}| + C$ 2. $\frac{1}{3} \sec^{-1} x^3 + C$ 3. $\log_e |1 - e^x| + C$

LEVEL III 1. $2\sqrt{\tan x} + C$ 2. $-\tan^{-1}(\cos x) + C$ 3. $\frac{\tan^2 x}{2} + \log_e |\tan x| + C$

(ii) Application of trigonometric function in integrals

LEVEL I 1. $-\frac{3}{4} \cos x + \frac{1}{12} \cos 3x + C$ 2. $\frac{1}{2} \left[x + \frac{\sin 6x}{6} \right] + C$

3. $\frac{x}{4} + \frac{1}{4} \sin 6x + \frac{1}{16} \sin 4x + \frac{1}{8} \sin 2x + C$

LEVEL II 1. $\frac{1}{4} \sec^4 x + C$ OR $\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + C$ 2. $\frac{2}{3} \sin 3x + 2 \sin x + C$

LEVEL III 1. $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$ 2. $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

(iii) Integration using Standard results

LEVEL I 1. $\frac{1}{2} \log_e \left| x + \frac{1}{2} \sqrt{4x^2 - 9} \right| + C$ 2. $\frac{1}{3} \tan^{-1} \left(\frac{x+1}{3} \right) + C$ 3. $\frac{1}{9} \tan^{-1} \left(\frac{3x+2}{3} \right) + C$

LEVEL II 1. $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2+1}{\sqrt{3}} \right) + C$ 2. $\tan^{-1}(\sin x + 2) + C$ 3. $\sin^{-1} \left(\frac{2x-1}{5} \right) + C$

LEVEL III

1. $\sin^{-1}\left(\frac{2x^2-1}{5}\right) + C$
2. $x + \log|x^2 - x + 1| + \frac{2}{\sqrt{3}} \log\left|\frac{2x-1}{\sqrt{3}}\right| + C$
3. $\sqrt{x^2 + 5x + 6} - \frac{1}{2} \log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + C$
4. $\sin^{-1} x + \sqrt{1-x^2} + C$ [Hint: Put $x = \cos 2\theta$]
5. $6\sqrt{x^2 - 9x + 20} + 34 \log\left|\left(\frac{2x-9}{2}\right) + \sqrt{x^2 - 9x + 20}\right| + C$

(iv) Integration using Partial Fraction

LEVEL I

1. $\frac{1}{3} \log(x+1) + \frac{5}{3} \log(x-2) + C$
2. $\frac{1}{2} \log(x-1) - 2 \log(x-2) + \frac{3}{2} \log(x-3) + C$
3. $\frac{11}{4} \log\left(\frac{x+1}{x+3}\right) + \frac{5}{2(x+1)} + C$

LEVEL II

1. $x - 11 \log(x-1) + 16 \log(x-2) + C$
2. $\frac{1}{4} \log x - \frac{1}{2x} + \frac{3}{4} \log(x+2) + C$
3. $\frac{3}{8} \log(x-1) - \frac{1}{2(x-1)} + \frac{5}{8} \log(x+3) + C$

LEVEL III

1. $\log(x+2) - \frac{1}{2} \log(x^2+4) + \tan^{-1} \frac{x}{2}$
2. $\frac{\log(1-\cos x)}{6} + \frac{\log(1+\cos x)}{2} - \frac{2 \log(1+2 \cos x)}{3} + C$
3. $\frac{1}{3} \log(1+x) - \frac{1}{6} \log(1-x+x^2) + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$ [Hint: Partial fractions]

(v) Integration by Parts

LEVEL I

1. $x \cdot \tan x + \log \cos x + C$
2. $x \log x - x + C$
3. $e^x \cdot \log \sec x + C$

LEVEL II

1. $x \sin^{-1} x + \sqrt{1-x^2} + C$
2. $\frac{x^3}{3} \sin^{-1} x + \frac{(x^2+2)\sqrt{1-x^2}}{9} + C$
3. $-\sqrt{1-x^2} \sin^{-1} x + x + C$
4. $2x \tan^{-1} x - \log(1+x^2) + C$
5. $\frac{1}{2} (\sec x \cdot \tan x + \log(\sec x + \tan x)) + C$

LEVEL III

1. $\frac{x}{2} [\cos(\log x) + \sin(\log x)] + C$
2. $\frac{e^x}{2+x} + C$ [Hint: $\int [e^x f(x) + f'(x)] dx = e^x f(x) + c$]

3. $\frac{x}{1+\log x} + C$

4. $e^x \cdot \tan x + C$
5. $\frac{e^{2x}}{13} (3 \sin 3x + 2 \cos 3x) + C$

(vi) Some Special Integrals

LEVEL I

1. $\frac{x\sqrt{4+x^2}}{2} + 2 \log|x + \sqrt{4+x^2}| + C$
2. $\frac{x\sqrt{1-4x^2}}{2} + \frac{1}{4} \sin^{-1} 2x + C$

LEVEL II 1. $\frac{(x+2)\sqrt{x^2+4x+6}}{2} + \log\left|(x+2) + \sqrt{x^2+4x+6}\right| + C$

2. $\frac{(x+2)\sqrt{1-4x-x^2}}{2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$

LEVEL III 1. $-\frac{1}{3}(1-x-x^2)^{3/2} + \frac{1}{8}(2x-1)\sqrt{1-x-x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right) + C$

2. $\frac{1}{3}(x^2+x)^{3/2} - \frac{11}{8}(2x+1)\sqrt{x^2+x} + \frac{11}{16}\log\left[(2x+1) + 2\sqrt{x^2+x}\right] + C$

(vii) Miscellaneous Questions

LEVEL II 1. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x - 1}{\sqrt{5}\tan x + 1}\right| + C$

2. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x + 1}{2\sqrt{2}}\right) + C$

3. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + C$

4. $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

5. $\tan^{-1}(\tan^2 x) + C$ [Hint: divide Nr. and Dr. by x^2]

6. $\frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2} + 4}{3}\right) + C$

LEVEL III 1. $-\frac{12}{13}x - \frac{5}{13}\log|3\cos x + 2\sin x| + C$

2. $\frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

3. $x + \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$

4. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + C$

5. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right| + C$

6. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2}\tan x}\right) + \frac{1}{2\sqrt{2}}\left|\frac{\tan x - \sqrt{2}\tan x + 1}{\tan x + \sqrt{2}\tan x + 1}\right| + C$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I 1. $\frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

2. $\frac{64}{231}$

3. $\left[\log\frac{3}{2} - 9\log\frac{5}{4}\right]$

LEVEL II 1. $5 + \frac{5}{2}\left[\log\frac{3}{2} - 9\log\frac{5}{4}\right]$

2. $\frac{e^2}{4}(e^2 - 2)$

(ii) Definite integrals as a limit of sum

LEVEL I 1. 6

2. 12

(iii) Properties of definite Integrals

LEVEL I 1. $\frac{\pi}{4}$

2. 1

3. $\frac{\pi}{4}$

LEVEL II 1. $\frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$

2. $\frac{\pi^2}{4}$

3. $\frac{\pi^2}{4}$

4. $\frac{\pi}{12}$

LEVEL III 1. $\frac{\pi}{2}$

2. $-\frac{\pi}{2} \log 2$

3. $\frac{\pi}{8} \log 2$

(iv) Integration of modulus function

LEVEL III 1. $\frac{19}{2}$

2. $\frac{11}{4}$

3. 4

Questions for self evaluation

1. $\log|x^2 + 3x - 18| - \frac{2}{3} \log\left|\frac{x-3}{x+6}\right| + c$

2. $-3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

3. $\left[\frac{1}{8} \left(3x + 2\sin 2x + \frac{\sin 4x}{4} \right) + c \right]$

4. $\tan^{-1}\left(1 + \tan \frac{x}{2}\right) + c$

5. $\frac{18}{15}x + \frac{1}{25} \log|3\sin x + 4\cos x| + c$

6. $x - \sqrt{1-x^2} \sin^{-1} x + c$

7. $\frac{64}{231}$

8. $\frac{3}{\pi} + \frac{1}{\pi^2}$

9. $-\frac{\pi}{2} \log 2$

10. 19/2