UNIT-VI

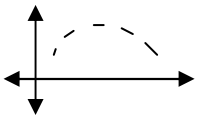## (CONTINUITY & DIFFERENTIABILITY)

1 MARKS QUESTION

1. Is real function $f(x) = \dfrac{1}{x}$ is continuous function. 　1

2. What is the value of continuous function $f(x)$ at $x=3$, $If \lim\limits_{x\to 3-} f(x) = K$? 　1

3. Give an example of a function which is continuous at a point but not differentiable at that point. 　1

4. If $f(x) = \dfrac{x^2-9}{x+3}$ is continuous function, then find the value of $f(3)$. 　1

5. If $f(x) = \begin{cases} x, & x<0 \\ -5, & x>0 \end{cases}$ for what value of m, $f(x)$ is continuous at $x=0$. 　1

6. For what value of $b, f(x) = \begin{cases} \dfrac{1}{b}, & x\neq 1 \\ 0, & x=1 \end{cases}$ is continuous at $x=1$? 　1

7. If possible, write an interval of $[x]$ such that $f(x)=[x]$ is continuous, where $[x]$ is greatest integer function. 　1

8. If f'(0)=5. Is f(x) is continuous at x = 0 ? 　1

9. If $y = \sin x^2$, find $\dfrac{dy}{dx}$ 　1

10. If $y = \log_x^x$. Find $\dfrac{dy}{dx}$ 　1

11. Differentiate $(\tan^{-1} x)^2$ W.r.t. $\tan^{-1} x$ 　1

12. Differentiate $\cos^{-1}(\dfrac{\pi}{2})$ w.r.t. x. 　1

13. If $y = \sin^{-1}(\dfrac{1}{\sqrt{2}})$. Find $\dfrac{dy}{dx}$ 　1

14. Is graph 　1

Shows that function is differentiable on R.

15. If $f(x) = \dfrac{|x|}{x}$. Find $f'(x)$ when $x>0$. 　1

16. Differentiate $e^{\sin x}$ w.r.t. $\sin x$. 　1

17. $If\ y\ is\ independent\ from\ x. What\ is\ \dfrac{dy}{dx}$? 　1

18. $Is\ x+y=5\ is\ implicit\ or\ \exp licit\ from\ of\ x$? 　1

19. $If\ u, v\ and\ w\ are\ function\ of\ x,\ then\ write\ product\ rule\ of\ differentiation,\ for$ 　1
$u.v.w,\ w.r.t.\ x.$

20. Differentiate $\tan(2x+3)$ w.r.t. x. 　1

4 MARKS QUESTIONS (Continuty)

1.   Discuss the continuity of the function $f(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ **2+2**

2.   Show that $f(x) = \begin{cases} 2x+1, & x \geq 2 \\ 5, & 0 < x < 2 \\ x^2 + 3, & x \leq 0 \end{cases}$ is continuous at $x = 2$.   **1+2+1**

3.   Show that every polynomial function is continuous.   **2+2**

4.   Determine whether f defined by $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ is a continuous function or not ?   **2+2**

5.   Find the value of k so that the given function is continuous at the given point   **2+2**

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & x \neq 0 \\ k+1, & x = 0\ at\ x = 0 \end{cases}.$$

6.   Let f be the function $f(n) = \dfrac{\sqrt{1+n}}{n}, n \neq 0$ **2+2**

   (a) $\lim\limits_{n \to 0} f(n)$

   (b) Is it possible to make it continuous

   (c) What is/ are the point of continuity

   (d) What choice, if any of $f(0)$ will make it continuous at x=0 ?

7.   Given that f(x) = [x] where [.] is the greatest integer function.   **2+2**
   (a)   Is $f(x)$ continuous at x?
   (b) Check the continuity of $f(x)$ at x = 25
   (c) What are the points of discontinuity of $f(x)$?

8.   If $y = \cot^{-1} \sqrt{\dfrac{1+\cos 2x}{1-\cos 2x}}$ $find$ $\dfrac{dy}{dx}$ **2+2**

9.   If $x = \sqrt{a^{\sin^{-1} t}} : y = \sqrt{a^{\cos^{-1} t}}$ show that $\dfrac{dy}{dx} = \dfrac{-y}{x}$ **2+2**

10.   Find $\dfrac{dy}{dx}$ **2+2**

   (a) $y = \cos\left(\sin \sqrt{ax+b}\right)$

   (b) $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

11.   If y + sin y = cos x   **2+2**
   Show that $\dfrac{dy}{dx} = \dfrac{-\sin x}{1+\cos x}$

12.   Find $\dfrac{dy}{dx}$ **2+2**

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

13. It   2+2

(a) $y = \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

(b) $y = \sin 2x - \sin 3x$ Find $\dfrac{dy}{dx}$

14. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$ 2+2

15. Find $\dfrac{dy}{dx}$ 2+2

    it $\begin{aligned}x &= \alpha(\theta + \sin\theta)\\ y &= \alpha(1-\cos\theta)\end{aligned}$

6 MARKS QUESTIONS

1. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in their ways mentioned below   2+2+2

   (a) By using product rule
   (b) By expanding the product to obtain a single polynomial
   (c) By logarithmic differentiation

2. For a positive constant 'a' where $y = a\left(t + \dfrac{1}{t}\right)$ and $x = \left(t + \dfrac{1}{t}\right)^a$ 2+2+2

   Find:

   (i) $\dfrac{dy}{dt}$

   (ii) $\dfrac{dx}{dt}$

   (iii) $\dfrac{dy}{dx}$

3. Find $\dfrac{dy}{dx}$ if $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

4. Find $\dfrac{dy}{dx}$ if $y^x + x^y + x^x = a^b$

5. If $(x-a)^2 + (y-b)^2 = c^2$ prove that $\dfrac{\left[1 + \left(\left(\dfrac{dy}{dx}\right)^2\right)\right]^{3/2}}{\dfrac{d^2 y}{dx^2}}$

   Is a constant independent of 'a' and 'b'

6. If $y = \left(\tan^{-1}x\right)^2$. Show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_2 = 2$

7. If $\cos y = x\cos(a+y)$ Prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$

8. Determine the value of a, b, c for which the function 1+2+2+1

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , x < 0 \\ c & , for\ x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & \end{cases}$$

for x >0

9. 6
   If the function f(x) defined by $f(x) = \begin{cases} \log\dfrac{(1+ax) - \log(1-bx)}{x} & if\ x \neq 0 \\ K & if\ x=0 \end{cases}$

   Is continuous at *x = 0 find K*

10 Find $\dfrac{dy}{dx}$, when 2+2+2

$$y = \left(x + \frac{1}{x}\right)^x + x\left(x\frac{1}{x}\right)$$